Assignment-1

Subject Code: BMAT0-201 Subject: Applied Mathematics-II

(UNIT-I)

1. Show that the radius of curvature of a circle is constant.
2. Prove that for the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, the radius of curvature is $\frac{a^2b^2}{p^3}$, where p is the perpendicular from the center to the tangent at (x,y).
3. If ρ_1 and ρ_2 are the radii of curvatures at the extremities of a focal chord of the parabola $y^2=4ax$, prove that $\rho_1^{-\frac{2}{3}}+\rho_2^{-\frac{2}{3}}=(2a)^{-\frac{2}{3}}$.
4. If ρ_1 and ρ_2 are the radii of curvatures at the extremities of two conjugate semi-diameters of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, show that $\left(\rho_1^{\frac{2}{3}}+\rho_2^{\frac{2}{3}}\right)(ab)^{\frac{2}{3}}=a^2+b^2$.
5. Show that for the catenary $y=\frac{c}{2}\left(e^{\frac{x}{c}}+e^{-\frac{x}{c}}\right)$, the radius of curvature at any point varies as square of it ordinate.
6. Show that radius of curvature at the end of the major axis of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is equal to the semi-latus rectum of the ellipse.
7. Show that radius of curvature at any point of the cardioid $r=a(1+\cos\theta)$ is $\frac{2}{3}\sqrt{2ar}$ and that $\frac{\rho^2}{r}$ is constant.
8. Show that the radii of curvature at the origin on the curve $x^3+y^3=3axy$ is equal to $\frac{3a}{2}$.
9. Show that the radius of curvature at any point of the cycloid $x=a(t+\sin t)$ and $y=a(1-\cos t)$ is $4a\cos\frac{t}{2}$.
10. Show that radius of curvature at any point of the cardioid $r=a(1-\cos\theta)$ varies as $\sqrt{r}$.

11. If $f(x) = ax^3 + 3bx^2$, determine a, b so that the graph of the function will have a point of inflexion at (-1, 2).

12. Examine the curve $y = x^3 - 9x^2 + 10x + 5$ for concavity upwards and concavity downwards.

13. Find the point of inflexion of the curve $y = \frac{1}{6}\left(x^3 - 6x^2 + 9x + 6\right)$.

14. Find the horizontal and vertical asymptotes of the curve $x^2y^2 - a^2\left(x^2 + y^2\right) = 0$. Also show that these form a square of side $2a$.

15. Find all the asymptotes of the curve $f(x, y) = y^3 - xy^2 - x^2y + x^3 + x^2 - y^2 - 1 = 0$.

16. Trace the curve $a^2y^2 = x^2\left(a^2 - x^2\right)$.

17. Trace the curve $y^2(2a - x) = x^3$ (Cissoid).

18. Trace the curve $x^3 + y^3 = 3axy$ (Folium of Descartes).

19. Trace the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$.

20. Trace the curve $x = a(\theta + \sin\theta), y = a(1 - \cos\theta)$.

21. Trace the curve $r = a(1 + \cos\theta), a > 0$.

22. Find the area enclosed by the curve $x(x^2 + y^2) = a(x^2 - y^2)$ and its asymptote.

23. In the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$, find the area between the curve and its base.

24. Find the area bounded by Cardioid $r = a(1 + \cos\theta)$.

25. Show that the area common to the Cardioid $r = a(1 - \cos\theta)$ and $r = a(1 + \cos\theta)$ is $\frac{a^2}{2}(3\pi - 8)$.

26. Find the area bounded by the parabola $y^2 = 4ax$, and its latus-ractum .

27. Find the area between parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

28. Find the length of the arc of the parabola $y^2 = 4ax$ cut off by the line $3y = 8x$.
29. Show that the length of an arc of the cycloid whose equations are $x = a(\theta - \sin\theta),\ \ y = a(1-\cos\theta)$ is $8a$.
30. Find the perimeter of the cardioid $r = a(1-\cos\theta)$. Also show that the upper half of the curve is bisected by the line $\theta = \dfrac{2\pi}{3}$.
31. Find the area of the loop of the curve $x = t^2$, $y = t(1-\dfrac{t^2}{3})$.
32. Find the volume and surface generated by the revolution of the Astroid $x = a\cos^3\theta,\ y = a\sin^3\theta$, about the $x-axis$.
33. Find the volume generated by the revolution of the area under one complete arch of the cycloid $x = a(\theta - \sin\theta),\ \ y = a(1-\cos\theta)$.
    The axis of revolution being (i) the $x-axis$ (ii) the $y-axis$.
34. Find the centre of gravity of a triangular lamina.
35. Find the centre of gravity of a solid in the form of a hemisphere of radius $a\ units$.
36. Find the centre of gravity of the area cut-off from the parabola $y^2 = 4ax$ by the line $y = mx$. Hence find the locus of the centroid.
37. Find the moment of inertia of a triangle about its base.
38. Find the moment of inertia of a solid sphere about its diameter.
39. Find the moment of inertia of hollow right circular cone about its axis.
40. Find the moment of inertia of the area intercepted by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ about the $x-axis$.

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